Space-time Trefftz-DG methods on tent pitching meshes for elastoacoustic wave propagation

Hélène Barucq, Henri Calandra, Julien Diaz, Elvira Shishenina

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1. EPC Magique 3D, Inria, E2S-UPPA, CNRS 2. Total E&P

PROS AND CONS

Adapted to the complex geometries

High-order accuracy and hp-adaptivity

Explicit semi-dicrete form

Conservation laws

Higher number of degrees of freedom,

compared to the methods with continuous approximation

E. TREFFTZ, 1926

Basis functions are **local solutions of the initial PDEs**

Frequency domain:

Farhat, Tezaur, Harari, Hetmaniuk (2003 - 2006), Gabard (2007), Badics (2014),
Hiptmair, Moiola, Perugia (2011 - 2016), Barucq, Bendali, Tordeux, Fares, Mattesi (2017)

Time domain:

Tsukerman, Egger, Kretzschmar, Schnepp, Weiland (2014 - 2015), Wang, Tezaur, Farhat (2014),
Moiola, Perugia (2016 - 2018), Banjai, Georgoulis, Lijoka (2017),
Perugia, Schöberl, Stocker, Wintersteiger (2019)

EXPECTED ADVANTAGES AND DRAWBACKS

- Higher order of convergence
- Flexibility in the choice of basis functions
- Low dispersion
- Incorporation of propagation directions in the discrete space
- Adaptivity and local space-time mesh refinement

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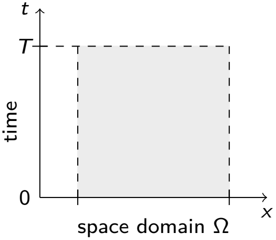
- Huge (sparse) global space-time matrix

01

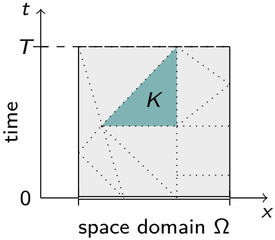
Mathematical Formulation

DOMAIN DESCRIPTION



$$\begin{cases} \dfrac{1}{c^2\rho}\dfrac{\partial p}{\partial t} + \mathrm{div}\, v = f & \text{in } \Omega \times [0, T], \\ \rho \dfrac{\partial v}{\partial t} + \nabla p = 0 & \text{in } \Omega \times [0, T], \\ & \\ v(\cdot, 0) = v_0, \;\; p(\cdot, 0) = p_0 & \text{in } \Omega, \\ v \cdot n^x = g_D & \text{in } \partial\Omega \times [0, T]. \end{cases}$$

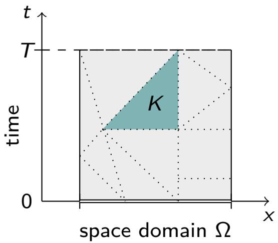
SPACE-TIME DISCRETIZATION



$$\mathcal{F}_h = \bigcup_{K \in \mathcal{T}_h} \partial K$$ - mesh skeleton

·····	$\mathcal{F}^I$	- internal
——	$\mathcal{F}^D$	- boundary
═	$\mathcal{F}^0$	- initial time
– –	$\mathcal{F}^T$	- final time

SPACE-TIME DISCRETIZATION



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$$(v_h, p_h) \in V^h(\mathcal{T}_h)^d \times V^h(\mathcal{T}_h) \equiv \boldsymbol{V}^h(\mathcal{T}_h)$$

$$(\omega, q) \in V^h(\mathcal{T}_h)^d \times V^h(\mathcal{T}_h) \equiv \boldsymbol{V}^h(\mathcal{T}_h)$$

SPACE-TIME DISCRETIZATION

$$-\sum_K \int_K p_h\left(\frac{1}{c^2\rho}\frac{\partial q}{\partial t} + \mathrm{div}\omega\right) + v_h \cdot \left(\rho\frac{\partial \omega}{\partial t} + \nabla q\right)$$

$$+\sum_K \int_{\partial K} \left(\frac{1}{c^2\rho}\breve{p}_h q + \breve{v}_h \cdot \omega\right) n_K^t + \left(q\hat{v}_h + \hat{v}_h\omega\right) \cdot n_K^x = \sum_K \int_K fq$$

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TREFFTZ SPACE

$$\mathbf{T}(\mathcal{T}_h) \equiv \left\{(\omega, q) \in \boldsymbol{V}^h(\mathcal{T}_h) \text{ s.t. } \frac{1}{c^2\rho}\frac{\partial q}{\partial t} + \mathrm{div}\omega \;=\; \rho\frac{\partial \omega}{\partial t} + \nabla q = 0,\;\; \forall K \in \mathcal{T}_h\right\}$$

SPACE-TIME DISCRETIZATION

$$-\sum_K \int_K p_h\left(\frac{1}{c^2\rho}\frac{\partial q}{\partial t} + \mathrm{div}\omega\right) + v_h \cdot \left(\rho\frac{\partial \omega}{\partial t} + \nabla q\right)$$

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SPACE-TIME DISCRETIZATION IN $\boldsymbol{T}$

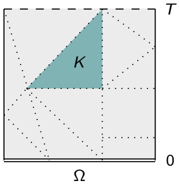
$$\sum_K \int_{\partial K} \left(\frac{1}{c^2\rho}\breve{p}_h q + \breve{v}_h \cdot \omega\right) n_K^t + \left(q\hat{v}_h + \hat{v}_h\omega\right) \cdot n_K^x = \sum_K \int_K fq$$

TREFFTZ-DG FORMULATION

$$\sum_K \int_{\partial K} \left(\frac{1}{c^2 \rho} \breve{p}_h q + \breve{v}_h \cdot \omega \right) n_K^t + \left(q \hat{v}_h + \hat{v}_h \omega \right) \cdot n_K^x = \sum_K \int_K fq$$

NUMERICAL FLUXES THROUGH THE ELEMENT FACES

$\begin{pmatrix} \hat{v}_h \\ \hat{p}_h \end{pmatrix}$	$\equiv$	$\begin{pmatrix} \{v_h\} + \beta [\![p_h]\!]_x \\ \{p_h\} + \alpha [\![v_h]\!]_x \end{pmatrix}$	on $\mathcal{F}^I$	····
$\begin{pmatrix} \breve{v}_h \\ \breve{p}_h \end{pmatrix}$	$\equiv$	$\begin{pmatrix} \{v_h\} \\ \{p_h\} \end{pmatrix}$	on $\mathcal{F}^I$	····
$\begin{pmatrix} \hat{v}_h \cdot n_K^x \\ \hat{p}_h \end{pmatrix}$	$\equiv$	$\begin{pmatrix} g_D \\ p_h + \alpha(v_h \cdot n_K^x - g_D) \end{pmatrix}$	on $\mathcal{F}^D$	—
$\begin{pmatrix} \breve{v}_h \\ \breve{p}_h \end{pmatrix}$	$\equiv$	$\begin{pmatrix} v_h \\ p_h \end{pmatrix}$	on $\mathcal{F}^T$	- -
$\begin{pmatrix} \breve{v}_h \\ \breve{p}_h \end{pmatrix}$	$\equiv$	$\begin{pmatrix} v_0 \\ p_0 \end{pmatrix}$	on $\mathcal{F}^0$	═



Inria

TREFFTZ-DG FORMULATION

Seek $(v_h, p_h) \in \boldsymbol{V}^h(\mathcal{T}_h)$ s.t. $\forall(\omega, q) \in \boldsymbol{T}(\mathcal{T}_h)$ it holds true:

$$\mathcal{A}_{TDG}\Big((v_h, p_h); (\omega, q)\Big) = \ell_{TDG}\Big(\omega, q\Big)$$

WELL POSEDNESS

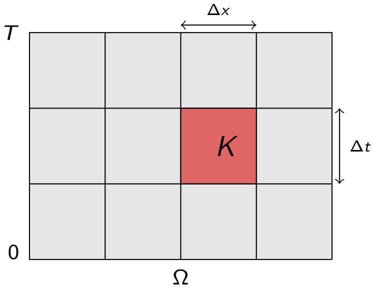
Trefftz-DG formulations for the first order Acoustic System, Elastodynamic System, and Elasto-Acoustic System are well-posed*.

* H. Barucq, H. Calandra, J. Diaz & Elvira Shishenina.
Space-Time Trefftz-Discontinuous Galerkin Approximation for Elasto-Acoustics. (2017)
https://hal.inria.fr/hal-01614126/document

02

Implementation

MESH AND ELEMENT NUMBERING



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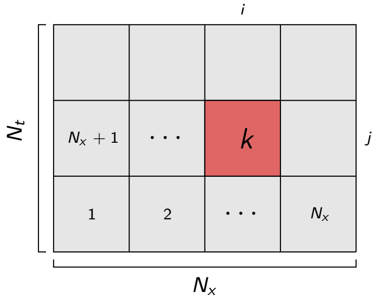
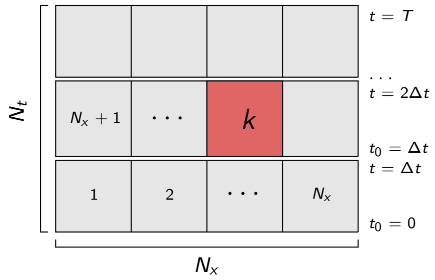


$$k = (j-1) \times N_x + i$$

MESH AND ELEMENT NUMBERING



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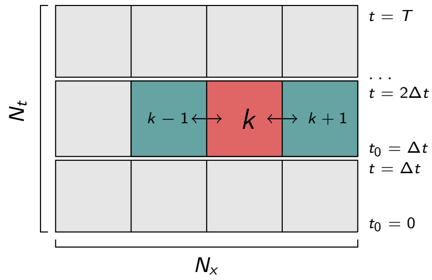

BILINEAR FORM

$$\mathcal{A}_{TDG} = \underbrace{\int_{\mathcal{F}^T} + \int_{\mathcal{F}^0}}_{\mathcal{A}^{\Omega}_{TDG}} + \underbrace{\int_{\mathcal{F}^I} + \int_{\mathcal{F}^D}}_{\mathcal{A}^{I}_{TDG}}$$

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GLOBAL SPACE-TIME MATRIX

$$\mathbf{M} = \bar{\Delta}_x M_{\Omega} + \bar{\Delta}_t M_I$$

BILINEAR FORM

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M_{Ω} - block-diagonal

BILINEAR FORM

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GLOBAL SPACE-TIME MATRIX

$$\mathbf{M} = \bar{\Delta}_x M_\Omega + \bar{\Delta}_t M_I$$

M_Ω - block-diagonal $\qquad$ M_I - sparse

GLOBAL MATRIX INVERSION

$$\mathbf{M}^{-1} \equiv \left[\bar{\Delta}_x M_\Omega + \bar{\Delta}_t M_I\right]^{-1}$$

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$$=$$

$$\left[I + \kappa\left({M_\Omega}^{-1} M_I\right)\right]^{-1}\left[\bar{\Delta}_x M_\Omega\right]^{-1}$$

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TAYLOR EXPANSION (SMALL ENOUGH$\kappa = \frac{\bar{\Delta}_t}{\bar{\Delta}_x} \propto \frac{\Delta t}{\Delta x}$)

$$\mathbf{M}^{-1} = \left[\sum_{n=0}^{\infty} (-1)^n \kappa^n \left({M_\Omega}^{-1} M_I\right)^n\right] \left[\bar{\Delta}_x M_\Omega\right]^{-1}$$

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GLOBAL MATRIX INVERSION

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block-diagonal x sparse

Approximate Inversion ($\kappa = 10^{-2}$)

n	$\Delta x = 10^{-2}$	$\Delta x = 2 \cdot 10^{-2}$	$\Delta x = 5 \cdot 10^{-2}$	$\Delta x = 10^{-1}$
3	1.4166E-05	4.3741E-05	2.8780E-04	2.5772E-03
4	3.1623E-07	1.2656E-06	5.3868E-05	1.2674E-03
5	2.8903E-07	9.1744E-07	4.1029E-05	1.3010E-03

Full Inversion ($\kappa = 10^{-2}$)

n	$\Delta x = 10^{-2}$	$\Delta x = 2 \cdot 10^{-2}$	$\Delta x = 5 \cdot 10^{-2}$	$\Delta x = 10^{-1}$
·	2.2540E-07	8.9583E-07	5.5811E-05	1.3004E-03

APPROXIMATE INVERSION ($\kappa = 10^{-2}$)

n	$\Delta x = 10^{-2}$	$\Delta x = 2 \cdot 10^{-2}$	$\Delta x = 5 \cdot 10^{-2}$	$\Delta x = 10^{-1}$
3	1.4166E-05	4.3741E-05	2.8780E-04	2.5772E-03
4	3.1623E-07	1.2656E-06	5.3868E-05	1.2674E-03
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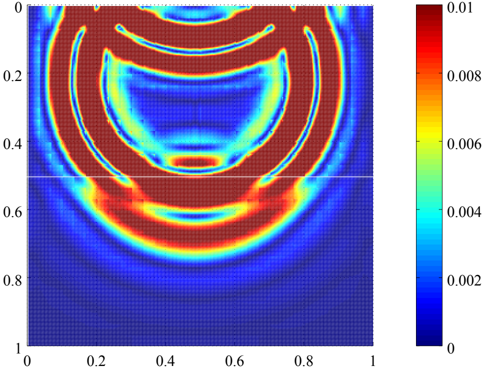
03

Numerical Results

2D Elasto-Acoustic System

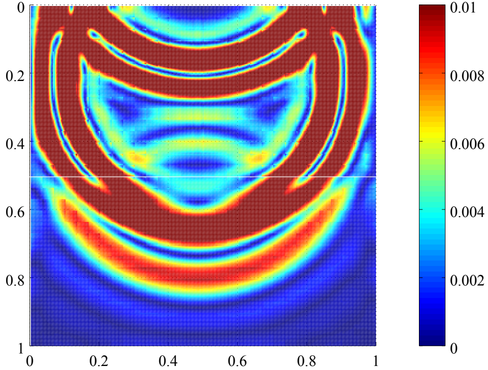
2D Elasto-acoustic system.

L^2 norm of numerical velocity v



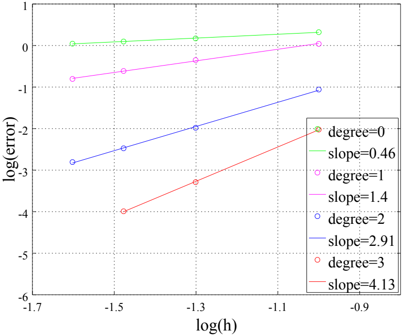
2D ELASTO-ACOUSTIC SYSTEM

2D Elasto-acoustic system.
L^2 norm of numerical velocity v

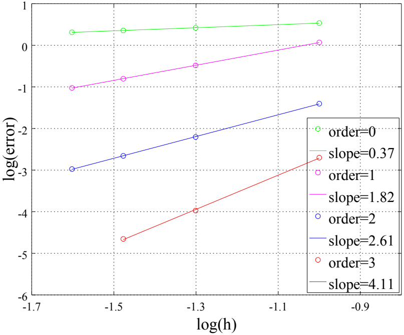


2D Acoustic and Elastodynamic Systems

2D Acoustic system.
Convergence of velocity v in function of cell size



2D Elastodynamic system.
Convergence of velocity v in function of cell size



04

Tent-Pitching Meshes

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A tent pitching scheme motivated by Friedrichs theory.(2015)

J. Gopalakrishnan, J. Schöberl, and C. Wintersteiger.
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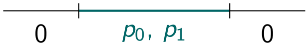
I. Perugia, J. Schöberl, P. Stocker and C. Wintersteiger.
Tent pitching and Trefftz-DG method for the acoustic wave equation.. (Preprint, 2019)

1D WAVE EQUATION

$$\left\{\begin{aligned}&\frac{1}{c_F^2}\frac{\partial^2 p}{\partial t^2}-\frac{\partial^2 p}{\partial x^2}=0,\\[2em]&p(\cdot,0)=p_0,\\&\frac{\partial p}{\partial t}(\cdot,0)=p_1.\end{aligned}\right.$$



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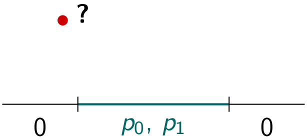

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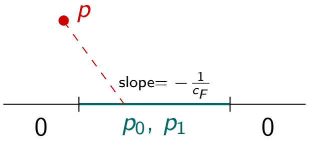

$$p(x,t) = \frac{1}{2}\left(p_0(x - c_F t) + p_0(x + c_F t)\right) + \frac{1}{2c_F}\int_{x-c_F t}^{x+c_F t} p_1(s)ds$$

Tent pitching meshes

1D WAVE EQUATION

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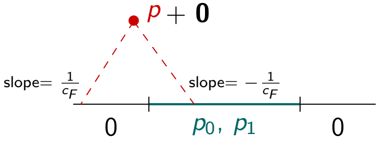

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1D Wave Equation

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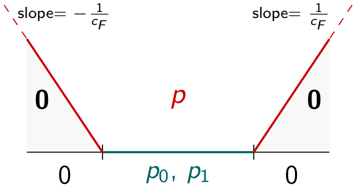

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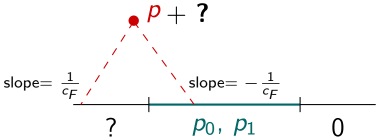

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MESH GENERATION

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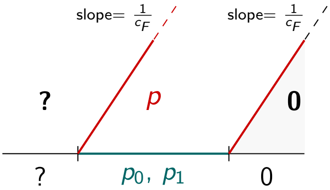

$$p(x,t) = \frac{1}{2}(p_0(x-c_F t) + p_0(x+c_F t)) + \frac{1}{2c_F}\int_{x-c_F t}^{x+c_F t} p_1(s)ds$$

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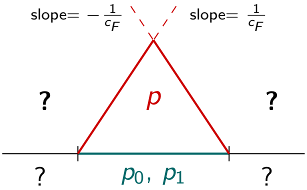
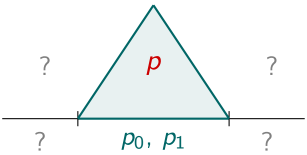
$$\begin{cases} \dfrac{1}{c_F^2}\dfrac{\partial^2 p}{\partial t^2} - \dfrac{\partial^2 p}{\partial x^2} = 0, \\ \\ p(\cdot, 0) = p_0, \\ \dfrac{\partial p}{\partial t}(\cdot, 0) = p_1. \end{cases}$$



$$p(x,t) = \frac{1}{2}\left(p_0(x - c_F t) + p_0(x + c_F t)\right) + \frac{1}{2c_F}\int_{x-c_F t}^{x+c_F t} p_1(s)ds$$

MESH GENERATION

$$\begin{cases} \dfrac{1}{c_F^2}\dfrac{\partial^2 p}{\partial t^2} - \dfrac{\partial^2 p}{\partial x^2} = 0, \\ \\ p(\cdot, 0) = p_0, \\ \dfrac{\partial p}{\partial t}(\cdot, 0) = p_1. \end{cases}$$

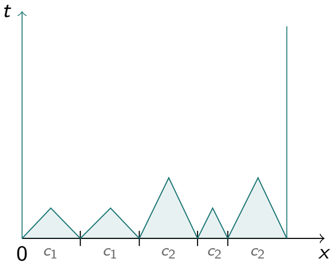


$$p(x,t) = \frac{1}{2}(p_0(x - c_F t) + p_0(x + c_F t)) + \frac{1}{2c_F}\int_{x-c_F t}^{x+c_F t} p_1(s)ds$$

MESH GENERATION

1^{st} step

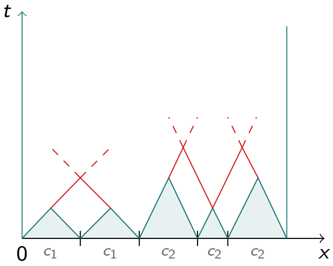
initial condition



MESH GENERATION

2^{nd} step

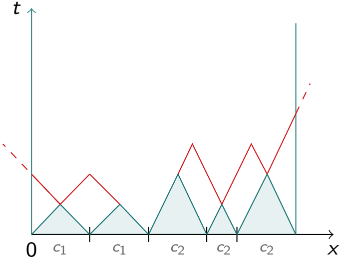
initial condition



MESH GENERATION

2^{nd} step

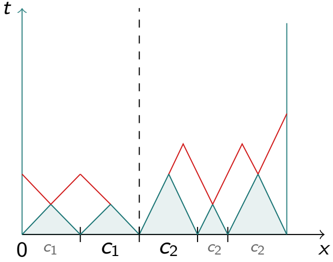
initial condition
boundary condition



MESH GENERATION

2^{nd} step

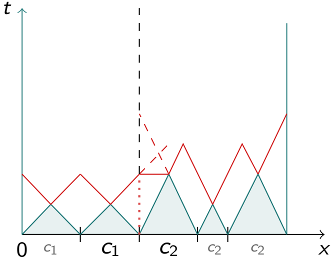
initial condition
boundary condition
heterogeneity?



MESH GENERATION

2^{nd} step

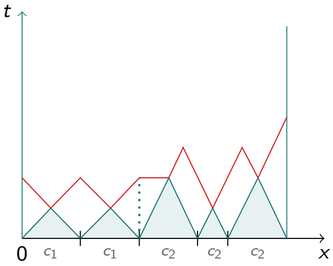
initial condition
boundary condition
heterogeneity?



MESH GENERATION

2^{nd} step

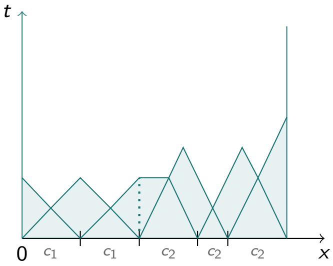
initial condition
boundary condition
heterogeneity



MESH GENERATION

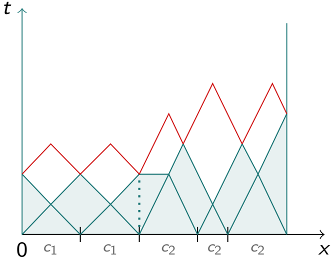
2^{nd} step

initial condition
boundary condition
heterogeneity



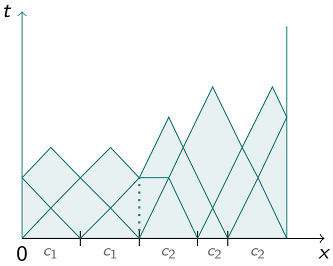
MESH GENERATION

and so on..



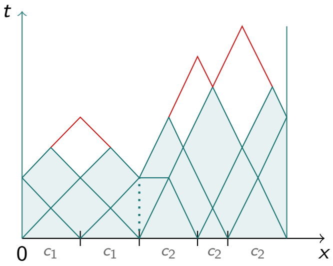
MESH GENERATION

and so on..



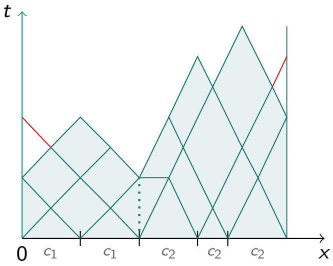
MESH GENERATION

and so on..



MESH GENERATION

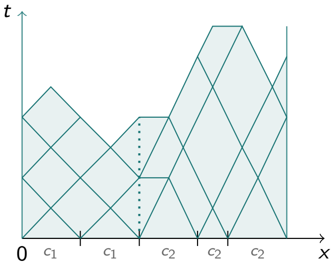
and so on..



MESH GENERATION

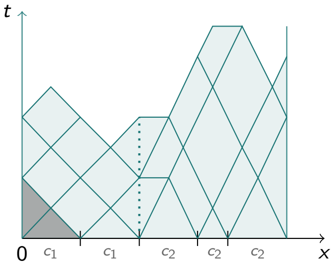
and so on..

$$c_F|n_K^x|/|n_K^t| \leq 1$$



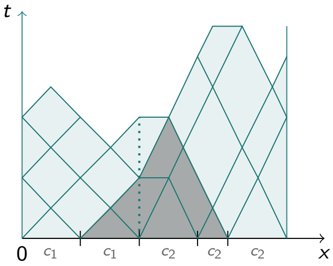
MESH GENERATION

mesh is divided in
space-time patches



MESH GENERATION

mesh is divided in
space-time patches



MESH GENERATION

mesh is divided in
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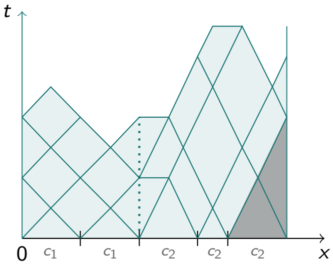
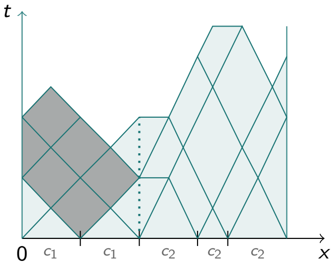


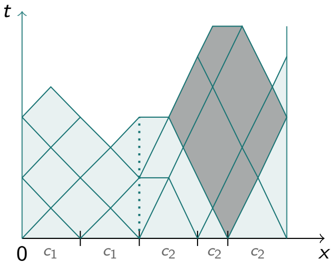
MESH GENERATION

mesh is divided in
space-time patches



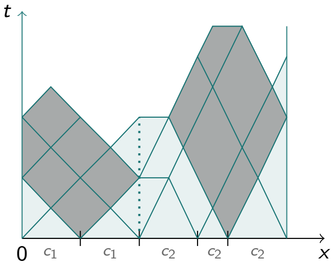
MESH GENERATION

mesh is divided in
space-time patches



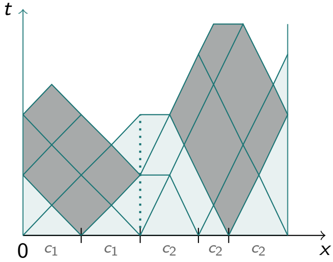
MESH GENERATION

mesh is divided in
space-time patches



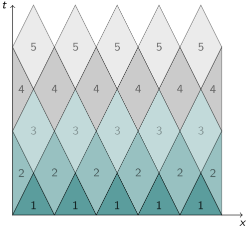
MESH GENERATION

mesh is divided in
space-time patches

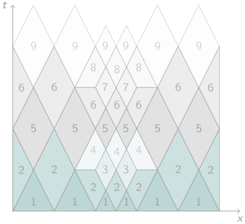


can be computed independently!

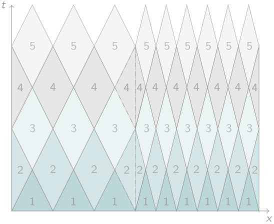
1D TENT PITCHING MESHES



Uniform tent mesh
(homogeneous medium)

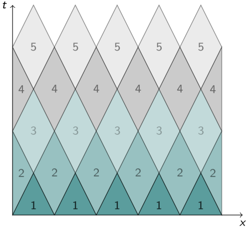


Non-uniform tent mesh
(homogeneous medium)



Non-uniform tent mesh
(heterogeneous medium)

1D TENT PITCHING MESHES



Uniform tent mesh
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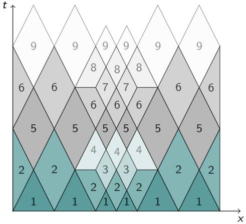
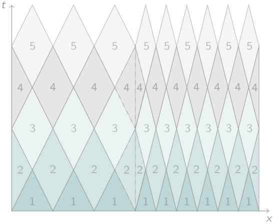
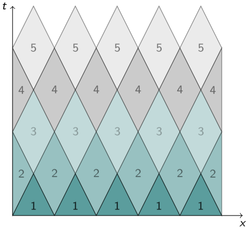


Non-uniform tent mesh
(homogeneous medium)

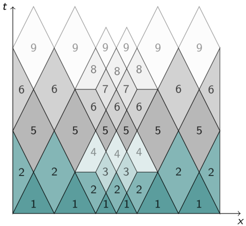


Non-uniform tent mesh
(heterogeneous medium)

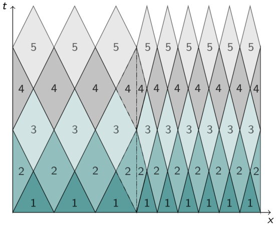
1D TENT PITCHING MESHES



Uniform tent mesh
(homogeneous medium)

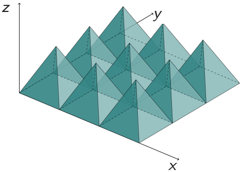


Non-uniform tent mesh
(homogeneous medium)

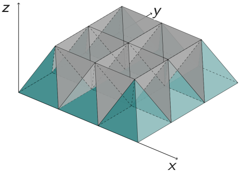


Non-uniform tent mesh
(heterogeneous medium)

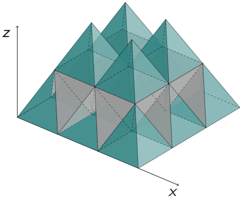
2D Tent Pitching Meshes



1^{st} step



2^{nd} step



3^{rd} step

STABILITY CONDITIONS

$c_F|n_K^x|/|n_K^t| \leq 1$ - TDG acoustic formulation

$V_P|n_K^x|/|n_K^t| \leq 1$ - TDG elastodynamic formulation

3D+TIME

Tent Pitcher &
Space-time DG

$\downarrow$

explicit scheme
local time-stepping

4D integrals

3D+TIME

Tent Pitcher &
Space-time DG

$\downarrow$

explicit scheme
local time-stepping

4D integrals

Space-time
Trefftz-DG

$\downarrow$

global
sparse matrix

3D integrals

3D+TIME

Tent Pitcher & Space-time DG	$\rightleftharpoons$	Space-time Trefftz-DG
$\downarrow$		$\downarrow$
explicit scheme local time-stepping		**global sparse matrix**
4D integrals		**3D integrals**

3D+TIME

Tent Pitcher &
Space-time Trefftz-DG

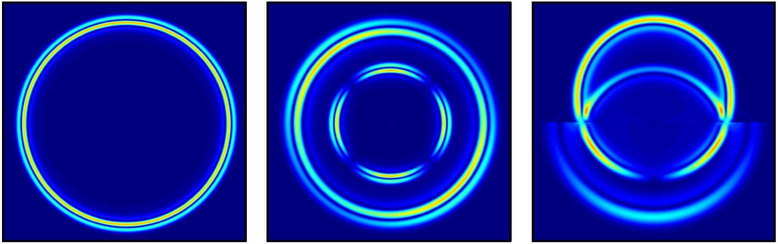
$\downarrow$

explicit scheme
local time-stepping

3D integrals

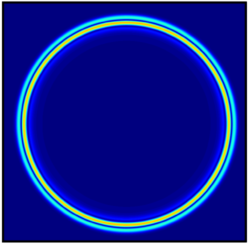
PROPAGATION OF NUMERICAL VELOCITY

2D Acoustic domain. 2D Elastodynamic domain. 2D Elasto-acoustic domain.



PROPAGATION OF NUMERICAL VELOCITY

2D Acoustic domain.



P-wave

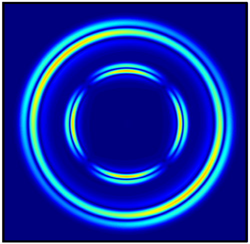
2D Elastodynamic domain.

2D Elasto-acoustic domain.

PROPAGATION OF NUMERICAL VELOCITY

2D Acoustic domain.

2D Elastodynamic domain.



P, S-waves

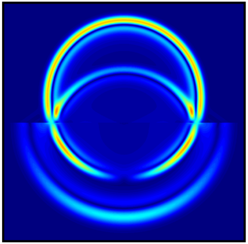
2D Elasto-acoustic domain.

Propagation of Numerical Velocity

2D Acoustic domain.

2D Elastodynamic domain.

2D Elasto-acoustic domain.



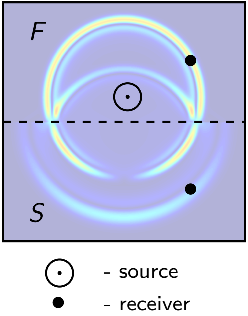
incident, reflected
P, S-waves,
P, S head waves

PROPAGATION OF NUMERICAL VELOCITY

2D Acoustic domain.

2D Elastodynamic domain.

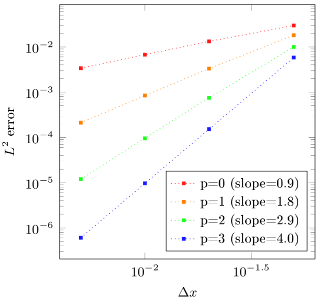
2D Elasto-acoustic domain.



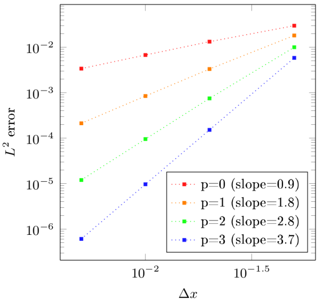
CONVERGENCE CURVES

Convergence of velocity v_F (left) and v_S (right) in space and time as a function of cell size $\Delta x = \Delta y = min\{V_P, c_F\}\sqrt{2}\Delta t$ (point-source case).

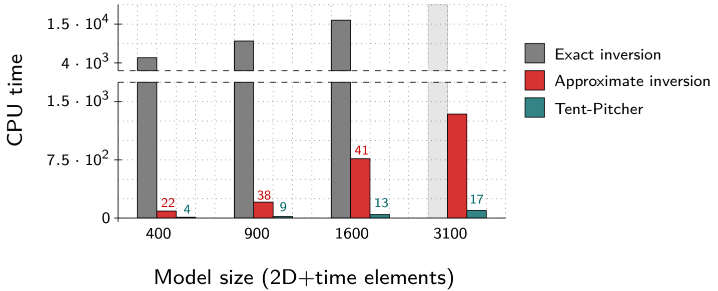
$\alpha_1 = \beta_1 = 0.5$



$\alpha_1 = \beta_1 = 0.5$



SPEEDUP FOR DIFFERENT MODEL SIZES



05

Conclusion

On-going Work and Perspectives

- Analytical study of convergence
- 4D Mesh generation [1]
- Absorbing boundary conditions
- Implicit-explicit coupling
- A posteriori error estimates, hp-adaptivity
- Space-time parallelization

[1] K. Voronin.
A parallel mesh generator in 3D/4D. (2018)